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ABSTRACT

The merger of two single-product manufacturers into a multi-product firm leads to the joint production of two goods which in the pre-merger industry configuration were produced separately. Given that, pre-merger, the two firms participate in different markets, the merger does not affect the number of firms per market. The merger is socially desirable if it does not cause product-specific marginal costs to rise. However, if firms are potential rivals, the merger offers them a more profitable but socially undesirable alternative. Efficiency gains from such a merger cannot compensate social losses from less potential competition.

Key words: Potential competition, economies of scope, cost-complementarity, cost-substitutability

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Introduction

In this paper, I study the case of a merger of two single-product manufacturers into a multiproduct firm.

In the pre-merger industry configuration, two single-product monopolists sell two different goods in two separate markets[1]. Firms produce positive outputs and earn non-negative profits. Thereafter, I assume that an exogenous change in the pre-merger environment gives single-product firms the possibility to choose whether to remain single-product or to become multiproduct. The decision of the two firms -or any of the two- to become multiproduct would lead the industry along other configurations, in which at least one of the two firms would participate in both markets. I study the impact of the alternative configurations on the decision of firms to merge. Like in the original industry configuration, firms are assumed to have positive sales and non-negative profits.

Higher total quantities produced in each market, under the assumption of a negative slope of the demand function, mean lower prices and higher levels of social welfare.

Multiproduct firms produce under technologies which are described in a multiproduct cost function, while the "stand alone" production of each of the two goods implies a single-product cost function.

The joint production of the two goods generates economies or diseconomies which are implied in the cost function in two different ways:

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1. The two markets are "separate" in the sense that the two goods are neither substitutes nor complements in consumer preferences. In terms of location models, the "distance" of the two markets is infinite.

1. Economies or diseconomies which affect the marginal costs of production due to production complementarity or substitutability between the two goods.

2. Fixed economies of scope due to subadditivity of fixed costs of joint production as compared to the fixed costs of the "stand-alone" production of the two goods.

The concepts of synergies [2] matching economies[3], and managerial ability[4], as used in the literature on mergers - although under various analytical specifications- correspond to the concept of cost complementarity used here. The concept of cost substitutability refers to the opposite[5].

The concept of fixed economies of scope coincides with Bianco's (1991) "fixed costs sharing" and Baumol's et al. (1982) "subadditivity of fixed costs".

Subadditivity of fixed costs together with cost complementarity leads to economies of scope over the whole range of production, while with cost substitutability, economies of scope are achieved only within a certain range of production. In either case, joint production is less costly and therefore, more efficient than the "stand-alone" production of the two goods. In case of joint production of the two goods, higher cost complementarity means more efficient production and higher cost substitutability means less efficient production. Indeed, at a firm's level efficiency is necessarily implied in the relevant cost function, since each firm is producing a bundle of goods at

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2. Farrell & Shapiro (1991).
3. Dutz (1991).
4. Bianco (1991).
5. In Salinger (1991), these magnitudes are used and modelled in a way, very similar to the one adopted here, although the paper refers to vertical mergers and the substitutability or complementarity of the goods are characteristics of demand and not of production.

the production frontier. At the industry level, an industry configuration is more efficient if it results in the joint production of complementary goods and it is less efficient if it results in the joint production of cost substitutes.

Farrell & Shapiro (1990) criticise the rule which is used in the literature to estimate the effect of a merger on the Herfindahl-Hirschman index (H). They note that "if all firms maintain their pre-merger market shares, the merger will not affect neither consumers nor nonparticipant firms, so it will be socially desirable if and only if it is privately profitable".They use an equilibrium analysis[6] to derive the post-merger market shares and then calculate the effect of the merger on H. Synergies are shown to play an important role in the effect that a merger will have on social welfare. Furthermore, in their **Proposition 2**, they show that **"if a merger generates no synergies, then it causes price to rise"**.

In **Section 2**, comparing the pre-merger and the post merger configurations, I show that Farrell & Shapiro's Proposition does not hold if the merger reduces the total number of players but not the number of firms per market. According to a common policy implication drawn from the literature on mergers,[7] the proponents of a merger should be able to prove that the merger will lead to synergies resulting from the combination of their complementary productive assets. I show that it is sufficient for the proponents of a merger -which does not affect the number of firms per market-

6. See also Salant et al. (1988).

7. "The burden of proof as to cost savings or other offsetting efficiencies however, should rest squarely on the proponents of a merger, and here I would require a very high standard [of proof]." Fischer (1987)

"Efficiencies are easy to promise, yet may be difficult to deliver. All mergers.....will tout 'synergies'." White (1987) to **prove** that the goods which they are going to produce jointly, **are not cost substitutes.**

In **Section 3,** I show that if one of the two firms is in the position to choose between multiproduct and single-product activity, it will not accept to merge with the other firm. Furthermore, the decision of the firm to become multiproduct leads to higher levels of social welfare.

If both firms are in the position to choose between single-product and multiproduct activity, firms are involved in a prisoner's dilemma. Simultaneous entry in each other's market makes both firms worse off and benefits consumers. However, firms may propose a merger in order to avoid the Nash equilibrium of the prisoner's dilemma. The anti-competitive character of such a merger is very difficult to prove, especially when it leads to higher levels of social welfare as compared to the pre-merger industry configuration.

As far as methodology is concerned, **Section 2** assesses the desirability of a merger on the basis of a comparison between the pre-merger and the post-merger industry structures. In **Section 3,** I compare the merger to other industry configurations which, under certain conditions, may result from the decision of firms to start selling in another market instead of merging. The argument developed in **Section 2** refers to the effect of a merger on actual competition, while the argument of **Section 3** refers to the effect of a merger on potential competition between the proponents of the merger. A merger policy based on the approach of **Section 3** would be less permissive than a policy based on a comparison of pre-merger and post-merger industry structures.

1. The Model

The markets for two goods, 1 and 2, are supplied by firms **A** and **B**. Depending on whether firms possess the 'know how' for the production of one of the two goods or both of them, and, consequently, on whether it is profitable for them to become multiproduct rather than to remain single-product, each firm may participate in one of the two markets or in both of them. Firms **A** and **B** are the only decision-making centers. Therefore, I do not allow for a potential entry of a new player. Furthermore, I do not allow for a potential exit of any of the two firms, since I assume that in all industry configurations, the equilibrium profits of both firms are non-negative. In that sense, the industry configurations studied here are not necessarily long-run equilibrium configurations.

Let q_{i_1} and q_{i_2} be the sales of firm i in markets 1 and 2 respectively, where i = **A, B.**

Total quantities in markets 1 and 2 are given respectively by

(1) $Q_1 = q_{A_1} + q_{B_1}$ and (2) $Q_2 = q_{A_2} + q_{B_2}$.

The inverse demand functions for products 1 and 2 are given respectively by

(3) $P_1 = a - Q_1$ and (4) $P_2 = c - Q_2$.

If firm i produces the two products jointly, production costs are given by

(5) $C_{12} = F_{12} + \beta_1 q_{i_1} + \beta_2 q_{i_2} + 2\beta\sqrt{q_{i_1} q_{i_2}}$ [8].

If firm i is a single-product manufacturer of product 1, production costs are given by

(6) $C_1 = F_1 + \beta_1 q_{i_1}$.

If it is a single-product manufacturer of product 2, costs are

(7) $C_2 = F_2 + \beta_2 q_{i_2}$.

with $\beta_1 > 0$, $\beta_2 > 0$, $q_{i_1} > 0$, $q_{i_2} > 0$, $F_1 \geq 0$, $F_2 \geq 0$, $F_{12} \geq 0$.

and $\dfrac{\beta}{2\sqrt{q_{i_1} q_{i_2}}} = \partial C_{12}/\partial q_1 \partial q_2$ denotes cost complementarity (if β is negative) or substitutability (if β is positive). For marginal costs of production to be non-negative, the absolute value of the parameter β must be small enough to satisfy

$$-\beta\sqrt{q_{i_1}/q_{i_2}} \leq \min\{\beta_1 , \beta_2\}.$$

The production of the two products in the same plant may lead to savings in the fixed costs of production. That is,

(8) $F_{12} \leq F_1 + F_2$.

The profit function for a multiproduct firm i is

(9) $\Pi_i = (a-Q_1)q_{i_1} + (c-Q_2)q_{i_2} - \beta_1 q_{i_1} - \beta_2 q_{i_2} - 2b\sqrt{q_{i_1} q_{i_2}} - F_{12}$.

8. For a further discussion of the properties of the cost function (5), see APPENDIX 1.

The profit function of single-product firm **A** in market 1 is given by

$$(10)\ \Pi_A = (a - Q_1)q_{A_1} - \beta_1 q_{A_1} - F_1 \ ,$$

and in market 2 by

$$(11)\ \Pi_B = (a - Q_2)q_{B_2} - \beta_2 q_{B_2} - F_2 \ .$$

For simplicity of the algebra, we will assume that subadditivity of the fixed costs of the joint production of the two products as compared to the fixed costs of the "stand-alone" production of each product, will always be of the type

$$(12)\ F_{12} = F_1 = F_2 = f \ ,$$

which is a special case of (8). The implication of this assumption is that fixed costs of production are the same for single-product and multiproduct firms. This can be the case where the fixed costs of production relate to expenditure for salaries of the administrative staff, rent and maintenance of the plant and in general, factors of production that are not affected by the decision of the firm to become multiproduct. Indeed, such a decision is more likely to involve sunk costs spent on R&D, a new licensing agreement, or the purchase of a new machine. In this paper, I ignore the role of sunk costs in the decision of firms to become multiproduct or to remain single-product. Any increase in the profits of a firm is assumed to provide it with an incentive, sufficient to justify any level of sunk costs.

The two markets are assumed to be of equal sizes. This is expressed in

(13) $M = a - \beta_1 = c - \beta_2$ [9].

In the pre-merger industry configuration, firm **A** is a monopolist in market 1 and firm **B** is a monopolist in market 2. We assume that such a pattern is dictated by the fact that each firm is the only possessor of the 'know how' for the production of the good that it is actually producing. Let this industry configuration be S_1 .

From equations (1) and (2) we get

(14) $Q_1=q_{A_1}$ and (15) $Q_2=q_{B_2}$

Firms set quantities to maximise their profits, so that

(16) $\partial\Pi_A/\partial q_{A_1}= \partial[(M-q_{A_1})q_{A_1} - f]/\partial q_{A_1} = 0$ and

(17) $\partial\Pi_B/\partial q_{B_2}= \partial[(M-q_{B_2})q_{B_2} - f]/\partial q_{B_2} = 0$.

From equations (16) and (17) we derive the quantities that the two monopolists produce in equilibrium

(18) $q_{A_1} = q_{B_2} = Q_1 = Q_2 = M/2$.

The profits of the two firms are given by

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9. For symmetric industry structures -in which each firm produces one of the two products or each firm produces both products- the implied assumption is that profitability in market 1 equals profitability in market 2.
For the interpretation of (13) as equality of the sizes of the two markets, see also Martin (1990).

$$(19) \quad \Pi_A = \Pi_B = \frac{M^2}{4} - f \ .$$

Firms earn non-negative profits if

$$(20) \quad 0 \leq f \leq M^2/4 \ .$$

2. The Merger of Two Single-Product Monopolists in a Multi-Product Firm

Under the assumptions of **Section 1**, firms cannot shift to multiproduct activity unless they reach an agreement for a merger.

I study here the case of a merger of the two single-product monopolists into a multiproduct firm. The merger does not affect actual competition, since its proponents are not rivals in the pre-merger industry structure.

Let the firm that results from the merger of firms **A** and **B** be denoted by **C** and the resulting industry configuration by S_2. Multiproduct technologies are implied in the cost function (5).

From equation (1) and (2) we get

$$(21) \quad Q_1 = q_{C_1} \qquad \text{and} \qquad (22) \quad Q_2 = q_{C_2} \ .$$

The resulting profit function is given by

$$(23) \quad \Pi_C = (M - q_{C_1})q_{C_1} + (M - q_{C_2})q_{C_2} - 2\beta\sqrt{q_{C_1}q_{C_2}} - f \ .$$

Firm **C** sets quantities in markets 1 and 2 to maximise its total profits, which, under the symmetry assumption, happens together with maximisation of the profits from each market separately. Furthermore, equal profitability in the two markets means that the firm will choose to produce equal quantities of each of the two products. This is expressed in

$$(24)\quad \partial\Pi_C/\partial q_{C_1} = M - 2q_{C_1} - \beta\sqrt{q_{C_2}/q_{C_1}} = 0 \;,$$

$$(25)\quad \partial\Pi_C/\partial q_{C_2} = M - 2q_{C_2} - \beta\sqrt{q_{C_1}/q_{C_2}} = 0 \quad \text{and}$$

$$(26)\quad q_{C_1} = q_{C_2} \;.$$

Simultaneous solution of equations (24), (25) and (26) gives the following equilibrium quantities

$$(27)\quad q_{C_1} = q_{C_2} = Q_1 = Q_2 = \frac{M-\beta}{2} \;.$$

The profits of the multiproduct monopolist are

$$(28)\quad \Pi_C = \frac{(M-\beta)^2}{2} - f \;.$$

For firm C to sell positive quantities in the two markets and earn non-negative profits, it must hold that

$$(29)\quad \beta < M \qquad \text{and} \qquad (30)\quad 0 \leq f \leq (M-\beta)^2/2 \;.$$

2.1. Profitability of the Merger

The two single-product firms will merge if their profits as a single multiproduct firm will be higher than the sum of their profits before the merger. This holds if $\Pi_{C/S_2} > \Pi_{A/S_1} + \Pi_{B/S_1}$, that is, when

$$(31) \qquad \frac{M^2}{2} - 2f < \frac{(M - \beta)^2}{2} - f \ .$$

Given that the merger will take place only if the two firms earn higher profits than in their pre-merger form, equation (31) is stronger than the restriction given by inequalty (30). Therefore, I solve inequality (31), taking into account only the restrictions given by (20) and (29).

The merger will take place for all values of the parameters β and f that satisfy (20), (29) and (31) simultaneously. This means that parameters β and f must satisfy simultaneously, conditions (i), (ii) and (iii) or conditions (i), (ii) and (iv), where

(i) $0 \leq f < M^2/4$,

(ii) $\beta < M$,

(iii) $\beta < M - \sqrt{M^2 - 2f}$ and (iv) $\beta > M + \sqrt{M^2 - 2f}$

Note that $0<M - \sqrt{M^2 - 2f} <0.29M$, while $1.57M<M +\sqrt{M^2 - 2f} <2M$ and that inequality (31) holds for $\beta=0$[10].

10. Graphs A2.1, A2.2 and A2.3 in Appendix 2 show the effect of an increase of M or a decrease of f on the critical value of β below which the merger will take place.

Conditions (i), (ii) and (iv) cannot hold simultaneously (note that $\beta < M$ and $0 \leq f < M^2/4$). Conditions (i), (ii) and (iii) hold for all values of the parameter β, for which

$$(32) \quad \beta < M - \sqrt{M^2 - 2f} .$$

I have assumed that **f** denotes fixed costs of production for both multiproduct and single-product manufacturers. As far as the merger is concerned, this implies that the fixed costs of production for firm **C** equal the fixed costs of production of each of the pre-merger firms **A** and **B**. In terms of inequality (32), this means that **f** refers to fixed economies of scope achieved by the merger of the two firms.

From inequality (32) we derive **Result 1**.

Result 1

(i) **If two single-product manufacturers of two different goods merge into a multiproduct firm and their merger does not lead to savings in the fixed costs of production (fixed economies of scope equal to zero), the merger will be profitable if and only if the two goods are complementary in production.**

(ii) **For every level of fixed economies of scope -other than zero- there are positive values of the parameter β (production substitutability), given by**

$$0 < \beta < M - \sqrt{M^2 - 2f} ,$$

for which the merger of the two single-product manufacturers into a multiproduct firm will be profitable.

Proof:

(i)

If $f = 0$, inequality (32) becomes $\beta < 0$.

(ii)

If $M^2/4 \geq f > 0$ then $M > \sqrt{M^2-2f}$ because $M^2 > M^2-2f$. Therefore, $0 < \beta < M - \sqrt{M^2 - 2f}$ defines the positive values of the parameter β for which the merger will be profitable for positive values of f.

Q.E.D.

2.2. Desirability of the Merger

In order to compare two different industry structures in terms of their effect on social welfare, I compare the total outputs produced. Higher output, under the assumption of a negative slope of the demand curve, results in a lower price, which is supposed to imply higher levels of social welfare.

The merger will improve social welfare if
$Q_1/_{S_1} = Q_2/_{S_1} < Q_1/_{S_2} = Q_2/_{S_2}$, which happens if

$$(33) \qquad \frac{M}{2} < \frac{M - \beta}{2} .$$

Inequality (33) holds for all β for which $\beta < 0$. If $\beta = 0$, pre-merger and post-merger quantities are equal and the merger does not have any effect on social welfare. The merger will cause prices to rise, if $\beta > 0$.

Result 2

A merger that generates no synergies (β=0) does not have any effect on prices if it does not affect the number of firms per market.

If the firm that results from the merger of two single-product manufacturers produces jointly two production complementary goods, the merger causes prices to fall, while if it produces production substitutes, the merger causes prices to rise.

In the case of additivity of fixed costs of joint production (when $F_1 + F_2 = F_{12}$ or $f = 0$) as compared to the fixed costs of stand-alone production, an unregulated merger induced by the expectation of an increased profit, leads to higher levels of social welfare. In that case, a privately profitable merger is socially desirable and viceversa.

The fact that if the two products are complementary the total market share and the total profits of the merging manufacturers increase should not be seen as an increase of market power, but rather as an incentive for them to make a decision that leads to more efficiency and higher social welfare.

Nevertheless, the two firms may decide to merge and produce jointly two cost-substitutable goods. This can happen when multiproduct activity leads to savings in the fixed costs of production. In that case, the merger is privately profitable but not desirable in terms of social welfare. Indeed, according to Result 1, if the fixed costs of a multiproduct firm are subadditive with respect to the fixed costs of two single-product firms, the merger will be profitable, even if it leads to joint production of production-substitutable goods. This will result in higher prices, and higher cost of production due to the inefficient co-existence of the two products in the same production line. Therefore, subadditivity of fixed costs may lead to a decision for an inefficient and socially undesirable merger which will cause prices to rise and resources to be spent inefficiently to cover the excessive costs due to substitutability between the two products.

If the interaction of the two products has no effect on the unit-costs of production and thereafter on prices (**Result 2**), a decision for a merger which is inspired by the fact that fixed costs of joint production are subadditive, leads to a Pareto superior outcome. Indeed, it leads to a position where firms are better off due to savings in the fixed costs of production and the consumers are not worse off since prices do not rise.

In terms of the model presented, the difference between my results and those obtained in Farrell & Shapiro is limited to the case in which the parameter β takes the value 0. That is, when the merger generates no synergies. At a general level, my conclusions are analogous to the conlusions reached in the literature.

The argument that a merger increases market power, is not relevant to the case studied here. The merger results in a reduction of the number of players but the number of firms per market is the same before and after the merger, since the pre-merger single-product firms participate in different markets. Such a merger has no effect whatsoever on actual competition.

I consider a policy maker with "lexicographic preferences" -in the sense that he wants first to make sure that the merger will not harm social welfare and only then to accept efficiency and the resulting increase of profitability as a good argument in favour of the proposed merger.

According to **Result 2**,

the policy maker should consider the merger as socially desirable even if it does not generate synergies, provided that:

1. The number of firms per market is not affected by the merger and

2. The joint production that is going to be undertaken by the merged firms does not increase the marginal costs of production.

The policy-maker must investigate whether the technologies of the joint production of the two products do not cause unit-costs of production and, thereafter, prices to rise.

When the two products do not generate synergies, a merger which leads to savings due to subadditivity of fixed costs and does not increase market power leads to a situation which is Pareto-superior to the pre-merger one.

3. The Merger of two Potential Rivals

In **Section 2,** each firm was constrained to produce only one of the two products. The merger of the two firms could not have any impact on potential competition. In this section, I analyse the effect of a merger on potential competition.

I assume an exogenous change in the pre-merger environment. Firms are not constrained to produce only one of the two products each.

3.1. The Proponents of the Merger are Potential Rivals in One of the Two Markets

I study first the case in which one of the two firms -firm **A**- is in the position to choose between multiproduct and single-product activity, while the other -firm **B**- cannot produce other than product 2. Let the industry configuration that results from the decision of firm **A** to become multiproduct be S_3. The configuration that would result if this option were available to firm **B** instead of firm **A** is denoted by S_3' and is symmetric to S_3.

In S_3, equations (1) and (2) become

(35) $Q_1 = q_{A_1}$ and (36) $Q_2 = q_{A_2} + q_{B_2}$

The profit functions of the two firms are expressed respectively in

(37) $\Pi_A = (M - q_{A_1})q_{A_1} + (M - q_{A_2} - q_{B_2})q_{A_2} - 2\beta\sqrt{q_{A_1} q_{A_2}}$

and

(38) $\Pi_B = (M - q_{A_2} - q_{B_2})q_{B_2}$.

Firm **A** acts as a monopolist in market 1. Firms **A** and **B** act as Cournot duopolists in market 2. Therefore, they set quantities to maximise their profits taking each other's sales as given. Equilibrium in the two markets is simultaneous and this implies simultaneous solution of equations

(39) $\partial\Pi_A/\partial q_{A_1} = M - 2q_{A_1} - \beta\sqrt{q_{A_2}/q_{A_1}} = 0$

(40) $\partial\Pi_A/\partial q_{A_2} = M - q_{B_2} - 2q_{A_2} - \beta\sqrt{q_{A_1}/q_{A_2}} = 0$

(41) $\partial\Pi_B/\partial q_{B_2} = M - q_{A_2} - 2q_{B_2} = 0$.

In order to simplify the algebra[11], I assume that in the problem of maximisation of the multiproduct firm, the effect of product 1 on production costs of product 2 is treated as equal to the effect of product 2 on production costs of product 1. This is expressed in

$$(42) \quad \beta\sqrt{q_{A_1}/q_{A_2}} \simeq \beta\sqrt{q_{A_2}/q_{A_1}} \simeq \beta \; ^{12}.$$

The implication of this assumption is that any difference in the optimal output levels of the two products of the multiproduct manufacturer will result from differences in the profitability of the two markets due to different numbers of firms per market.

The system of equations (39)-(41) reduces to the following system of equations:

$$(43) \quad \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} \begin{vmatrix} q_{A_1} \\ q_{A_2} \\ q_{B_2} \end{vmatrix} = \begin{vmatrix} M-\beta \\ M-\beta \\ M \end{vmatrix}$$

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11. The non-linear functional form of equations (39) and (40) yields difficulties both in mathematical operations and in the comparison of the results obtained in this section to the other results obtained in this paper.
12. This does not mean that firm **A** will produce equal quantities of the two products, since the number of firms in market **1** is not equal to the number of firms in market **2** and, in general, firm **A** will tend to sell more in market **1** than in market **2**.
When $\beta = 0$, firm **A** will sell $q_{A_1} = M/2$ in market 1 and $q_{B_1} = M/3$ in market **2**. In that case, the assumption expressed by equation (42), does not affect the solution of the system of equations (39), (40) and (41). The assumption above, is not very restrictive for small values of the parameter β.
In a more general model that would allow for a number of firms of each type, the assumption becomes less restrictive for large numbers of firms per market.

Solving (43), I obtain the levels of outputs that firms sell in each market. The total quantity sold in market 1 is produced by firm **A** and is given by

$$(44)\quad q_{A_1} = \frac{M - \beta}{2} = Q_1 \ .$$

The sales of firm **A** in market **2** are

$$(45)\quad q_{A_2} = \frac{M - 2\beta}{3}$$

and the sales of firm **B** are

$$(46)\quad q_{B_2} = \frac{M + \beta}{3} \ .$$

The total quantity of product 2 sold by the two firms is

$$(47)\quad Q_2 = \frac{2M - \beta}{3} \ .$$

The profits of firm **B** are

$$(48)\quad \Pi_B = \frac{(M+\beta)^2}{9} - f \ .$$

In the calculation of the profits of firm **A**, I approximate the geometric mean of q_{A_1} and q_{A_2} with their arithmetic mean[13].

Equilibrium profits for firm **A** are given by

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13. When $\beta=0$, the arithmetic mean of $q_{A_1}= M/2$ and $q_{A_2}= M/3$ is 0.416 and their geometric mean is 0.408.

$$(49)\ \Pi_A = \frac{(M-\beta)^2}{4} + \frac{(M-2\beta)^2}{9} - f\ .$$

For firms to have positive sales in each market and earn non-negative profits, and therefore for the industry configuration S_3 to be non-trivial[14], it must hold that

$$(50)\quad -M < \beta < M/2\ ,$$

$$(51)\quad 0 \leq f \leq \frac{(M-\beta)^2}{4} + \frac{(M-2\beta)^2}{9}$$

and

$$(52)\quad 0 \leq f \leq \frac{(M+\beta)^2}{9}\ .$$

Firm **A** will become multiproduct if its profits in S_3 are higher than its profits in S_1. This happens if $\Pi_{A/S_3} > \Pi_{A/S_1}$ or, applying equations (19) and (49), if

$$(53)\quad \frac{(M-\beta)^2}{4} + \frac{(M-2\beta)^2}{9} - f > \frac{M^2}{4} - f\ .$$

The industry configuration S_3 will not be reached if firm **A** does not obtain profits at least as high as in configuration S_1. Therefore, the condition of profitability of configuration S_3 for firm **A** (inequality (53)) is stronger than conditions (20) and (51). Let -M/2 be an arbitrarily chosen lower bound for the values of the parameter β. Inequalities (50) and (52) become

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14. Note that if any of the firms had zero sales in any of the makets or if any of the firms were earning negative profits, and therefore, had to exit from the industry, studying S_3 would not make any sense.

(50') $-M/2 < \beta < M/2$ and

(52') $0 \leq f \leq M^2/36$ which also satisfies (20).

I solve (53) subject to (50') and (52'). Firm **A** finds it profitable to become multiproduct if

(54) $-M/2 < \beta < 0.13M$.

Firm **B** earns lower profits than in $\mathbf{S_1}$ for all values of the parameter β that satisfy (50') and (52'), since

(55) $\frac{(M+\beta)^2}{9} - f < \frac{M^2}{4} - f$ for $\beta < M/2$.

Finally, considering the sum of quantities of products **1** and **2** as a measure for social welfare, the decision of firm **A** to become multiproduct will be socially desirable if

(56) $\frac{M - \beta}{2} + \frac{2M - \beta}{3} \geq M$

that is, when

(56') $\beta \leq 0.2M$

From inequalities (54), (55) and (56'), I derive the following result:

Result 3

If the decision of firm A to enter into the market of product 2 is profitable, it leads to higher levels of social welfare.

Following such a decision, firm B, which is the incumbent in the market of product 2, earns lower profits.

We see that firm **A** may find it profitable to produce jointly two production substitutable goods and that the resulting configuration will be socially desirable, while firm **B** will always experience losses from the entry of firm **A** in market **2.** In that sense, **Result 3** implies that private or social losses due to inefficiencies resulting from the joint production of two production substitutable goods, will be off-set by gains from multiproduct activity and the effect of an increase in the number of firms per market on social welfare.

3.1.1. Profitability of the Merger

Instead of becoming multiproduct, firm **A** could merge with firm **B.** This decision will depend on whether its share of profits in firm **C** is higher than those it can earn as either a single-product manufacturer in S_1, or a multiproduct manufacturer in S_3. I assume that after the merger, firm **A** earns half the profits of firm **C**, since its market share and its profits in the pre-merger industry configuration are equal to the market share and the profits of its potential partner, firm **B.** Firm **A** will prefer to merge with firm **B,** rather than to become multiproduct if

$$(57) \quad \frac{(M-\beta)^2}{4} + \frac{(M-2\beta)^2}{9} - f < \frac{(M-\beta)^2}{4} - \frac{f}{2} \quad .$$

Solution of (57) gives

$$(58) \quad \beta > \frac{M-3\sqrt{f/2}}{2} \; .$$

Under the assumption that the industry configuration S_3 is non-trivial, as implied in inequalities (50') and (52'), we reach the following result:

Result 4

If firm A is the single-product manufacturer of product 1 with the possibility of entering into the market of product 2, and firm B is the single-product manufacturer of product 2 without the possibility of producing any other product, firm A will never find it profitable to merge with firm B.

Proof:

Firm **A** will prefer the merger to its pre-merger single-product form if

(i) $\beta < M - \sqrt{M^2 - 2f}$ (due to inequality (32))

and, in order to find it more profitable to merge than to become multiproduct, it must hold that

(ii) $\beta > \frac{M-3\sqrt{f/2}}{2}$ (due to inequality (58)) .

But for all values of $0 \leq f \leq M^2/36$, it is true that

$$M - \sqrt{M^2 - 2f} < \frac{M-3\sqrt{f/2}}{2}$$

which means that conditions (i) and (ii) can never hold simultaneously. Therefore, for all values of β and f for which S_1, S_2 and S_3 are non-trivial, firm **A** will prefer to become multiproduct or to maintain its pre-merger single-product form, rather than to merge with firm **B**. Q.E.D.

Firm **B** would prefer to merge with firm **A** than to be a single-product firm in S_3, if it holds that $\Pi_{B/S_3} < \Pi_{C/S_2}/2$ or

$$(59)\quad \frac{(M+\beta)^2}{9} - f < \frac{(M-\beta)^2}{4} - f/2 .$$

Inequality (59) is satisfied for all values of β for which

$$(60)\quad \beta < \frac{26M - \beta\sqrt{676M^2 - 20(5M^2+18f)}}{10} .$$

However, there are no values of the parameter f from the interval $[0, M^2/36]$ that can satisfy the system of inequalities (58) and (60). Therefore, the merger of the two firms will not take place, not only because of unwillingness of firm **A**, but also because of incompatibility of the interests of firms **A** and **B**. This does not add anything to what we find in **Result 4**, since the merger has to be decided by both firms.

When one of the two firms is in the position to choose between multiproduct and single-product activity, the merger of the two firms will never take place. This conclusion might be less strict, if we had allowed for a negotiation between the two pre-merger firms concerning their shares in the profits of the post-merger firm **C**.[15] However, it is plausible to assume that firms **A** and **B** will have equal shares in the profits of firm **C**, since they earn equal profits in their single-product form before the merger.

15. It might be the case that firm **A** would accept to merge with firm **B** if it were offered more than half of the profits of the post-merger firm **C**, since we have seen that firm **B** experiences losses from the entry of firm **A** in market 2 (inequality (55)). This would lead our analysis to the study of a bargaining Nash solution, in order to determine whether there can be an agreement concerning the shares of the two pre-merger firms in the post-merger firm **C**.

3.1.2. Desirability of the Merger

In **Result 3**, we have seen that the decision of firm **A** to become multiproduct will be socially desirable if it is privately profitable. In **Result 4** we have seen that firm **A** will not find it profitable to merge with firm **B**, if it has the possibility to choose between single-product and multiproduct activity. I show that a merger would harm social welfare, if one of its proponents had the possibility of becoming multiproduct.

Indeed, if the two firms could reach an agreement for a merger, the quantity of the two products in the post-merger configuration would be lower than the quantity produced in $\mathbf{S_3}$. This is true for all values of the parameter β, for which configuration $\mathbf{S_3}$ is non-trivial, since $Q_1/_{S_2}+Q_2/_{S_2} \geq Q_1/_{S_3}+Q_2/_{S_3}$ or

$$(61) \qquad M - \beta \geq \frac{M - \beta}{2} + \frac{2M - \beta}{3}$$

holds only if $\beta \leq -M$, which would not be compatible with inequality (50'). Therefore we derive **Result 5**.

Result 5

If the two single-product firms that merge are potential rivals in one of the two markets, their merger in a multiproduct firm will harm social welfare.

If **Result 4** holds, the merger of firms **A** and **B** will never take place. However, I derive **Result 5** because it is obtained under less restrictive assumptions than **Result 4**.

3.2. The Proponents of the Merger are Potential Rivals in Both Markets

I now study the merger of firms **A** and **B** under the assumption that both of them are in the position to choose between multiproduct and single-product activity. Each of the two firms possesses the 'know how' for the production of both products. Starting from the industry configuration S_1, firm **A** can enter the market of product **2** and firm **B** the market of product **1**. Firms are involved in a game, in which they choose between becoming multiproduct (strategy **"MULT"**) or remaining single-product (strategy **"SINGL"**). If both firms decide to become multiproduct, the resulting industry configuration is S_4. The outcome of the game determines whether configuration S_4 is reached or not.

In S_4 , firms **A** and **B** behave as Cournot duopolists in the markets 1 and 2. Each firm sets quantities to maximise its total profits, taking the sales of its rival as given. The profit function of firm i is given in equation (9). Maximisation of profits with respect to each product gives the first order conditions

$$(62)\quad \partial\Pi_i/\partial q_{i_1} = M - q_{j_1} - 2q_{i_1} - \beta\sqrt{q_{i_2}/q_{i_1}} = 0$$

and

$$(63)\quad \partial\Pi_i/\partial q_{i_2} = M - q_{j_2} - 2q_{i_2} - \beta\sqrt{q_{i_1}/q_{i_2}} = 0 \quad ,$$

where $\{i, j\} = \{A, B\}$.

Given that the industry structure in S_4 is symmetric (since we have identical firms, equal sizes of the two markets, and equal numbers of firms per market), each firm will produce equal quantities of the two goods. Therefore the system of equations (62) and (63) is satisfied by the solutions of the system of equations

$$(62')\quad \partial\Pi_i/\partial q_{i_1} = M - q_{j_1} - 2q_{i_1} - \beta = 0$$

and

$$(63')\quad \partial\Pi_i/\partial q_{i_2} = M - q_{j_2} - 2q_{i_2} - \beta = 0 \quad ,$$

where $\{i, j\} = \{A, B\}$. The solution of the system of equations (62') and (63') gives us the sales of each firm in each market,

$$(64)\quad q_{A_1} = q_{A_2} = q_{B_1} = q_{B_2} = \frac{(M-\beta)}{3} \quad .$$

Summation of the sales of the two firms per market gives us the total quantity produced for each market,

$$(65)\quad Q_1 = Q_2 = \frac{2(M-\beta)}{3} \quad .$$

The profits of each firm are given by

$$(66)\quad \Pi_A = \Pi_B = \frac{2(M-\beta)^2}{9} - f \quad .$$

For the industry configuration S_4 to be non-trivial, firms must have positive sales and non-negative profits. Therefore, it must hold that $\beta > -M$ and $f \leq \frac{2(M-\beta)^2}{9}$. Both conditions are satisfied by the values of the parameters β and f that satisfy conditions (50') and (52'). Therefore we do not need to impose further restrictions on the two parameters.

Firms may decide to become multiproduct or not, according to the outcome of "the game of entries". Each firm chooses its strategy from the set $\Omega = \{$"SINGL", "MULT"$\}$. The general payoff matrix of the game is

		Firm B	
		SINGL	MULT
Firm A	SINGL	$\Pi_{A/S_1} - f, \Pi_{B/S_1} - f$	$\Pi_{A/S_3'} - f, \Pi_{B/S_3'} - f$
	MULT	$\Pi_{A/S_3} - f, \Pi_{B/S_3} - f$	$\Pi_{A/S_4} - f, \Pi_{B/S_4} - f$

According to inequality (54) firms will not have an incentive to deviate from ("SINGL", "SINGL") if $\beta > 0.13M$. In that case the configuration S_1 is the Nash equilibrium of the game.

If $\beta < 0.13M$, each firm finds it profitable to become multiproduct. Furthermore, if a firm remains single-product when the other firm becomes multiproduct, it earns lower profits, not only with respect to the original industry configuration S_1 (see inequality (55)), but also with respect to what it would earn in S_4. Indeed, $\Pi_{B/S_3} < \Pi_{B/S_4}$ holds if

$$(67) \quad \frac{(M+\beta)^2}{9} - f < \frac{2(M-\beta)^2}{9} - f ,$$

which is true if

$$(67') \quad \beta < 0.17M .$$

Given that each firm wants to deviate from ("SINGL", "SINGL") for all values of the parameter β for which $\beta < 0.13M$, inequality (67') leads me to **Result 6.**

Result 6

If firms A and B sell in two different, equally profitable markets and if the two firms are potential entrants in each other's market, each firm prefers competition in both markets to competition in only one of them, if it finds competition in one market more profitable than no competition at all.

Therefore, for $\beta < 0.13M$, the Nash equilibrum of the game is ("MULT", "MULT").

I compare the profits of the two firms before and after the simultaneous entry in each other's market. Firms earn more in S_4 than in S_1 if $\Pi_{i/S_4} > \Pi_{i/S_1}$ or

$$(68) \quad \frac{2(M-\beta)^2}{9} - f > \frac{M^2}{4} - f .$$

Inequality (67) holds for $\beta > -0.06M$. Therefore, if the two products are cost complementary, economies of joint production may offset firms' losses from increased competition.

If $-0.06M < \beta < 0.13M$, firms are involved in a prisoner's dilemma, since they enter in each other's market and they earn lower profits than in S_1.

The simultaneous entry of the two firms into each other's market will lead to higher levels of social welfare if $Q_{1/S_1} = Q_{2/S_1} < Q_{1/S_4} = Q_{2/S_4}$ or

$$(69) \quad \frac{M}{2} < \frac{2(M-\beta)}{3} ,$$

which holds if $\beta < 0.25M$. From (68) and (69), I derive **Result 7.**

Result 7

(i) There is a range of negative values of the parameter β ($\beta < -0.06M$) for which the decision of the two single-product firms to become multiproduct, entering into each other's market, leads to a Pareto superior outcome, where both consumers and producers are better off.

(ii) There is a range of values of the parameter β -both negative and positive- ($-0.06M< \beta < 0.13M$) for which the decision of the two single-product firms to become multiproduct, entering into each other's market, is socially desirable but not privately profitable.

(iii) There is a range of positive values of the parameter β ($0.13M<\beta < 0.25M$) for which the decision of the two firms to remain single-product, is privately profitable but not socially desirable.

(iv) There is a range of positive values of the parameter β ($0.25M<\beta$) for which the decision of two firms to remain single-product, is privately and socially desirable.

The ranges of values of the parameter β, for which **Results 7(i)-7(iv)** hold, depend on the assumptions of the model. Nevertheless, the order of the results **7(i)-7(iv)** would be the same for all β and $f \geq 0$, for which S_1, S_3 and S_4 are non-trivial configurations.

3.2.1. Profitability of the Merger

If firms realise that a simultaneous entry in each other's market leads to lower profits than their profits in S_1, they may propose a merger as a way to avoid the Nash equilibrium of the prisoner's dilemma in which they are involved. In that case, they compare their profits in S_2 with the profits that they would earn in S_4. The merger will take place if $\Pi_{C/S_2}/2 > \Pi_{i/S_4}$ or

$$(70) \quad \frac{(M-\beta)^2}{4} - \frac{f}{2} > \frac{2(M-\beta)^2}{9} - f .$$

Inequality (70) holds for all values of β that satisfy conditions (50') and (52'). From inequality (70), I obtain **Proposition 1.**

<u>**Proposition 1**</u>

If the two single-product manufacturers A and B are potential entrants in each other's market, the merger of the two manufacturers into a multiproduct firm will always be more profitable than the entry of each firm in the other firm's market.

<u>**Proof:**</u>

$$\frac{2(M-\beta)^2}{9} - \frac{(M-\beta)^2}{4} < \frac{f}{2} \quad \text{or}$$

$$(M-\beta)^2 \left(\frac{2}{9} - \frac{1}{4}\right) < \frac{f}{2}$$ holds for all $f \geq 0$. Q.E.D.

From inequality (32), we get that if $\beta < M - \sqrt{M^2 - 2f}$, the two firms find it more profitable to merge than to remain single-product in S_1. From **Results 7(iii)** and **7(iv)** we know that if $\beta>0.13M$, firms will remain single-product although they have the possibility to enter into each other's market. Then according to **Result 8,** firms will never decide to merge, if the Nash equilibrium of "the game of entries" is **("SINGL", "SINGL").**

Result 8

If the two single-product firms A and B are potential entrants in each other's market and the Nash equilibrium of "the game of entries", which determines whether the two firms will become multiproduct or not, is given by ("SINGL", "SINGL"), then the two firms will never find it profitable to merge.

Proof:

The Nash equilibrium of the "game of entries" is given by ("SINGL", "SINGL") if and only if

(i) $\beta > 0.13M$.

Firms will find it more profitable to merge than to remain single-product if

(ii) $\beta < M - \sqrt{M^2 - 2f}$.

Conditions (i) and (ii) do not hold simultaneously, since it is true that

$$M - \sqrt{M^2 - 2f} < \frac{2M-\sqrt{4M^2-8(M^2/36)}}{2} = 0.03M < 0.13M$$

for all values of f, for which it holds that $0 \leq f \leq M^2/36$.

Q.E.D.

3.2.2. Desirability of the Merger

According to **Result 8**, if the Nash equilibrium of "the game of entries" is ("SINGL", "SINGL"), the merger will not take place. According to **Result 2**, the decision of firms **A** and **B** not to merge if products 1 and 2 are substitutes in production, is socially desirable if it is privately profitable.

The merger of the two firms will never lead to higher levels of social welfare if the two firms are potential entrants in each other's market and the Nash equilibrium of the "game of entries" is ("MULT", "MULT"). In that case, the merger would be socially desirable if $Q_1/S_2 = Q_2/S_2 > Q_1/S_4 = Q_2/S_4$ or

$$(71) \quad \frac{M - \beta}{2} > \frac{2(M - \beta)}{3}$$

which does not hold for those values of the parameter β, that satisfy inequalities (50') and (52'). Combining inequality (71), **Result 7, Proposition 1** and **Result 8**, I obtain **Result 9**.

Result 9

If two single-product firms A and B are potential entrants in each other's market, and if

(i) $\beta < 0$,

the decision of the two firms to merge rather than to become multiproduct is privately profitable (Proposition 1). In terms of social welfare, such a decision leads to a superior outcome as compared to the pre-merger industry structure (Result 2), but to an inferior outcome as compared to a simultaneous entry of the two firms in each other's market (inequality (71)).

(ii) $0 < \beta < 0.13M$,

the decision of the two firms to merge rather than to become multiproduct is privately profitable (Proposition 1). In terms of social welfare, the result of such a decision is inferior to the pre-merger industry structure (Result 2), as well as to

a simultaneous entry of the two firms in each other's market (inequality (71)).

(iii) 0.13M< β < 0.25M

the decision of the two firms to maintain their pre-merger, single-product form is privately profitable (Result 8). In terms of social welfare, the result of such a decision is inferior to a simultaneous entry of the two firms in each other's market (inequality (68)).

(iv) 0.25M< β,

the decision of the two firms to maintain their pre-merger, single-product form is privately profitable (Result 8). In terms of social welfare, the result of such a decision is superior to a simultaneous entry of the two firms in each other's market (inequality (68)).

Comparing **Result 2** to results **9(i)** and **9(ii)**, we see that if the two firms are potential rivals in both markets, there is a broader range of values of the parameter **β**, for which their merger is profitable, than if the two firms were not potential rivals. Furthermore, private profitability and social desirability are compatible only for some of the values of the parameter β, for which the two firms prefer their single-product, pre-merger form, than a merger or a simultaneous entry into each other's market (**Result 9(iv)**).

Therefore, if the proponents of a merger are potential rivals in both markets, the merger will never be socially desirable. The two firms propose the merger in order to increase their joint profits with respect to what they would earn if they decided to enter simultaneously in each other's market. The effect of such a merger on potential competition and social welfare is always negative. However, according to **Result 2**, if β < 0, the merger leads to higher levels of social welfare as compared to the pre-merger industry structure. The implication of this observation is that if the policy-maker ignored the fact that the proponents of a merger are potential rivals, the merger of the two single-product manufacturers into a multi-product firm would be regarded as an improvement of social welfare.

4. Conclusions and Policy Implications

Two single-product producers of two different products are the only potential entrants in each other's market. The two markets are of equal sizes. Firms, in all configurations that result from the decision of one or both of them to enter into the other firm's market, have positive sales and earn non-negative profits. Under these assumptions, I have studied the effect of a merger on potential competition and on social welfare.

In the pre-merger industry structure the two single-product firms sell in two different markets. The merger of the two single-product manufacturers into one multiproduct firm reduces the number of players but not the number of firms per market.

In **Section 2**, the two firms do not have the possibility to choose between single-product and multiproduct activity. Therefore, the merger of the two firms does not affect potential competition.

With respect to the literature that deals with the trade off between efficiency and competition, following a merger, **Result 2** calls for a less strict merger policy than what Farrel & Shapiro (1990) suggests, since a merger which generates no synergies can be socially desirable, if it does not affect the number of firms per market and does not cause marginal costs of production to rise.

According to **Result 1**, if the fixed costs of the multiproduct cost function are subadditive with respect to the fixed costs of the "stand-alone" production, two single-product manufacturers of two production substitutes may find it profitable to merge into a multiproduct firm. We have seen in **Result 2** that this would lead to lower levels of social welfare. Therefore, I obtain a positive answer to the question whether a merger policy is necessary or not.

In **Section 3**, the proponents of the merger are potential rivals in one of the two markets or in both of them.

The merger of the two firms will always lead to lower levels of social welfare than the decision of one of the two firms or both of them to enter into the other firm's market. According to **Result 4**, a merger will not take place if there is only one firm that can choose between multiproduct and single-product activity and that firm is offered half the profits of the post-merger multiproduct firm. However, we see in **Results 5, 8, 9** and in **Proposition 1** that a merger between potential rivals will never be socially desirable if it is privately profitable. If two firms are potential rivals, they propose a merger in order to increase their joint profits[16]. Such a merger should be regarded as the result of the anti-competitive behaviour of its proponents as

In Bael & Bellis (1990) it is argued that "not every merger effected by a company holding a dominant position is to be regarded as an abuse". It is suggested that the policy maker, should take into account the effect of the merger on actual and potential competition and the resulting advantages of consumers from technical progress. The "case-handler" should have the right to carry out any kind of investigation and firms should pay heavy fines if they provide misleading information.

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16. This, following Phlips (1988), is equivalent to an agreement between the two firms to "move towards the profit frontier".

We have seen that if the merger of two single-product manufacturers into a single-product firm leads to joint production of cost-complementary products, marginal costs of production fall and consumers are better off. However, consumer's advantage from technical progress is offset by losses due to the negative effect of the merger on potential competition. If the proponents of a merger are potential competitors, the merger is not socially desirable even if it promises efficiencies.

The policy implications drawn refer to industries in which multi-product activity is subject to availability of the "know-how" for the production of more than one product and to the decision of manufacturers to enter in other markets. Therefore, the policy-maker should investigate whether the proponents of a merger could become multi-product instead of merging into a multi-product firm. The importance of any information on whether the proponents of a merger are potential rivals, is vital.

Appendix 1

Discussion of the properties of the cost function in (5)

In Figure A1 we see the shape of the multiproduct cost function given by equation (5).

Let a number $k \geq 0$ denote a ray of expansion of the production of the two goods, along which the ratio of outputs of goods 1 and 2 is constant, so that $k = q_{i_1}/q_{i_2}$. Along any ray of expansion, the multiproduct cost function (5) behaves as a single-product cost function with constant marginal costs. Indeed if we substitute q_{i_1} in (5) with kq_{i_2}, we get

$$C_{12} = F_{12} + \beta_1 kq_{i_2} + \beta_2 q_{i_2} + 2\ \beta\sqrt{kq_{i_2}^2} \quad \text{or}$$

$$(1.1)\quad C_{12} = F_{12} + (\beta_1 k + \beta_2 + 2\ \beta\sqrt{k}\ )q_{i_2}\ ,$$

the marginal cost of this linear function is non-negative if

$$(1.2)\quad \beta_1 k + \beta_2 \geq 2\ \beta\sqrt{k}\ .$$

For marginal product-specific costs to be non-negative at any point $(q_{i_{10}}, q_{i_{20}})$ of the plane $q_{i_1}0q_{i_2}$, it must hold that

$$\partial C_{12}/\partial q_{i_1} = \beta_1 + \beta\sqrt{q_{i_{20}}/q_{i_{10}}} \geq 0 \quad \text{and}$$

$$\partial C_{12}/\partial q_{i_2} = \beta_2 + \beta\sqrt{q_{i_{10}}/q_{i_{20}}} \geq 0$$

which are satisfied simultaneously by

$$(1.3)\quad -\beta \leq \min\{\beta_1\sqrt{q_{i_{10}}/q_{i_{20}}},\ \beta_2\sqrt{q_{i_{20}}/q_{i_{10}}}\}\ .$$

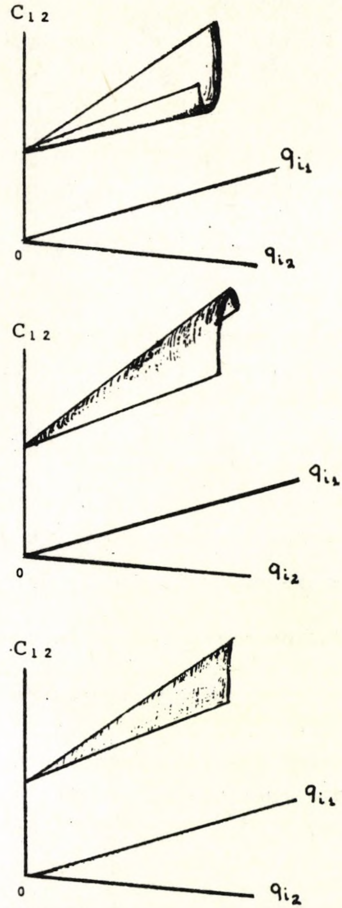
Conditions (1.2) and (1.3) do not require any restrictions on the parameter β, if $\beta \geq 0$. If $\beta < 0$, then for every ray of expansion k_0, the absolute value of β should be sufficiently small to satisfy (1.2) and (1.3). The two conditions become more restrictive for those points (q_{i_1}, q_{i_2}) that are very close to the axes $0q_{i_1}$ or $0q_{i_2}$ and the absolute value of β must be restricted to be very low.

For points that lie near the ray defined by k=1, or if $q_{i_1} \approx q_{i_2}$, conditions (1.2) and (1.3) are satisfied if

$$(1.4)\quad -\beta \leq \min\{\beta_1\ ,\ \beta_2\}\ .$$

Under this assumption, the geometric mean of the two outputs given by $\sqrt{q_{i_1}q_{i_2}}$ can be approximated by the arithmetic mean given by $(q_{i_1}+q_{i_2})/2$. Furthermore the effect of each product on the marginal cost of the other can be approximated by the parameter β alone.

This property makes the cost function given by (5) a useful tool for modeling as well as estimating cost complementaritiy or cost substitutability.



<u>Figure A1:</u> The cost function given by (5) for $\beta<0$, $\beta>0$ and $\beta=0$.

Appendix 2

The Graphical Solution of Inequality (31)

In Figures A2.1, A2.2 and A2.3 we see that the critical value of the parameter β below which the merger will take place depends on the values of **M** and of f.

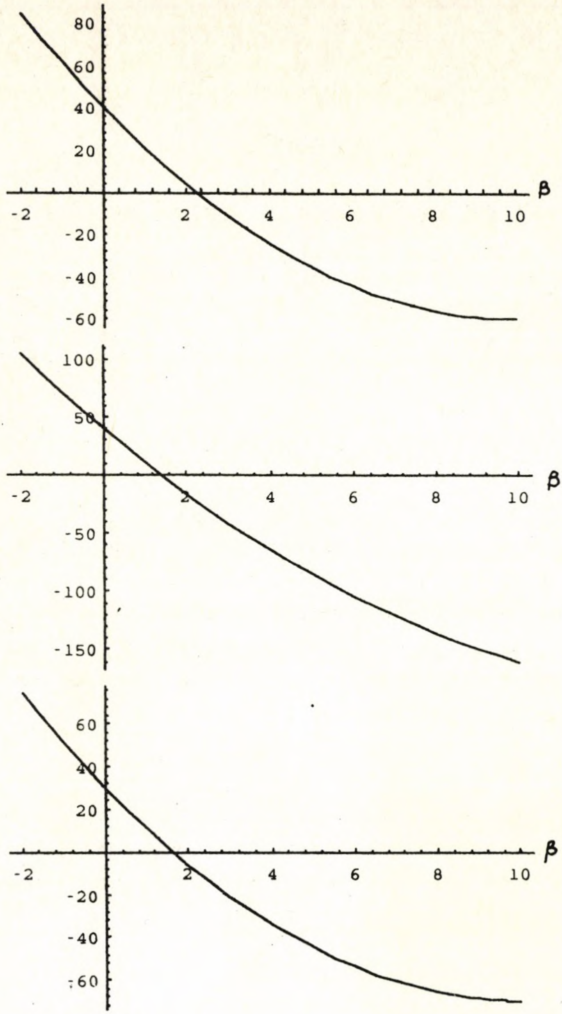
When

(2.1) $\beta^2+2f-2\beta M > 0$

inequality (31) is satisfied and the merger is profitable.

I plot the first part of (2.1) for M=10, f=20 in the interval (-2, 10) for the values of the parameter β (Fig. A2.1). I plot the same function for a higher value of **M** (M=15 - Fig. A2.2) and for a lower value of f (f=15 - Fig. A2.3). We observe that in both cases the critical value of the parameter β below which the merger will take place, decreases. In other words, if the merger achieves low savings in the fixed costs of production, or pofitability from each market is high, the firms will tolerate less inefficiency that results from the joint production of two substitutable products.

A further remark is that in the interval of values that are permitted for the parameter β, there is only one root for the quadratic expression (2.1) and consequently an upper bound for the values of β, for which the merger occurs.



A2.1: M=10
f=20

A2.2: M=15
f=20

A2.3: M=10
f=15

<u>**Figure A2**</u>: $\beta^2+2f-2\beta M > 0$. The critical value of the parameter β below which the merger takes place, decreases for lower savings in the fixed costs of production, or for higher profitability from each market.

REFERENCES

Bael, I. and Bellis, J.-F., (1990), Competition Law of the E.E.C., 2nd Edition, CCH Editions Ltd. Tax Business and Law Publishers.

Baumol, W. and Braunstein, Y., (1977), "Empirical Study of Scale Economies and Production Complementarity: The Case of Journal Publication", Journal of Political Economy, vol. 85, No.5, pp. 1037-1048.

Baumol, W., Willig, R. and Panzar, J., (1982), Contestable Markets and the Theory of Industry Structure, San Diego Harcourt, Brace, Jovanovich.

Bianco, M., (1991), "Diversification and Performance", Banca d'Italia, mimeo.

Bulow, J., Geanakoplos J. and Klemperer P., (1985), "Multimarket Oligopoly: Strategic Substitutes and Complements", Journal of Political Economy, vol 93, no. 3, pp. 488-511.

Caves, D., Christensen, L. and Tretheway, M., (1979), "Flexible Cost Functions for Multiproduct Firms", Review of Economics and Statistics, pp. 477-481.

Diewert, E., (1971), "An Application of the Shepherd Duality Theorem: A Generalized Leontief Production Function", Journal of Political Economy, vol. 79, pp. 481-507.

Dixit, A., (1984), "International Trade Policy for Oligopolistic Industries", Economic Journal, vol. 94, pp. 1-16.

Dixit, A. and Stiglitz, J., (1977), "Monopolistic Competition and Optimum Product Diversity", American Economic Review, vol. 67, pp. 297-308.

Dutz, M., (1991), "Horizontal Mergers, Matching Economies and International Integration", Princeton University and the World Bank, mimeo.

Farrell, J. and Shapiro, C., (1990), "Horizontal Mergers: An Equilibrium Analysis", American Economic Review, vol. 80, No 1, pp. 107-126.

Fischer, F., (1987), "Horizontal Mergers: Triage and Treatment" Journal of Economic Perspectives, 1, pp. 23-40.

Gabszewicz, J. and Thoron, S., (1991), "Gains and Losses from Horizontal Mergers." CORE Discussion Paper, No 9108.

George, K., (1989), "Do we Need a Merger Policy?" in J. Fairburn and J. Kay (eds), Mergers & Merger Policy, Oxford University Press.

Jacquemin, A. and Slade, M., (1990), "Cartels, Collusion and Horizontal Mergers" in R. Schmalensee and R. Willig, Handbook of Industrial Organisation, vol. 1, pp. 415-474.

Martin, S., (1990), Seminar Notes for the Course of Industrial Economics, E.U.I. Florence (Forthcoming Publication as an "Advanced Textbook of Industrial Economics").

Martin, S., (1988), Industrial Economics- Economic Analysis and Public Policy.

Phlips, L., (1988), The Economics of Imperfect Information, Cambridge University Press.

Salant, S., Sheldon, S. and Reynolds R., (1988), "Losses from Horizontal Mergers: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", in A. Daughty (ed) Cournot Oligopoly-Characterisation and Applications, Cambridge University Press.

Salinger, M., (1991), "Vertical Mergers in Multi-Product Industries and Edgeworth's Paradox of Taxation", Journal of Industrial Economics, vol. 39, No 5, pp. 545-556.

Salop, S., (1979), "Monopolistic Competition with Outside Goods", Bell Journal of Economics, vol.10, pp. 141-156.

Telser, L., (1991), "Industry Total Cost Functions and the Status of the Core." Journal of Industrial Economics, vol. XXXIX, No. 3, pp. 225-240.

White, L., (1987), "Antitrust and Merger Policy. A Review and Critique", Journal of Economic Perspectives, 1, pp. 13-22.

Willig, R., (1991), "Merger Analysis, Industrial Organisation Theory and Merger Guidelines", Brooking Papers: Microeconomics.

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